Hagedorn and Brown

Correlation overview

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What is the Hagedorn and Brown correlation?

• Multi-phase flow correlation

• Typically used to calculate the VLP curves for the OIL wells

What is the VLP curve?

Vertical Lift Performance Curve:

relates how much pressure is required to lift the fluid to the surface
How do we use the VLP curve?

Nodal Analysis:
How to calculate the VLP curve?

\[ \frac{dp}{dh} = \left( \frac{dp}{dh} \right)_{\text{GRAVITY}} + \left( \frac{dp}{dh} \right)_{\text{FRICTION}} + \left( \frac{dp}{dh} \right)_{\text{KINETIC}} \]
Hagedorn and Brown Math & Physics

\[
144 \frac{\Delta p}{\Delta h} = \bar{\rho}_m + \frac{f q_L^2 M^2}{2.9652 \times 10^{11} D^5 \rho_m} + \bar{\rho}_m \frac{\Delta \left( \frac{v_m^2}{2g_c} \right)}{\Delta h}
\]

\[
\bar{\rho}_m = \rho_L H_L + \rho_g (1 - H_L)
\]
Hagedorn and Brown - $H_L$

\[ (N_{LV}/N_{LV}^{0.575})(\frac{P}{P_a})^{0.10} \]

Hagedorn and Brown quick facts

- Test well in Dallas, TX
- Nominal pipe D: 1, 1¼, 1½ in
- Pipe length 1500 ft
- $q_L$ was varied
- For each $q_L$, GOR was varied as well
- Pressure gradient was measured
- 475 tests & 2905 pressure points
- Published in 1965

How it works?

For each $q_L$ [0 to AOF]

$\frac{dp}{dh} = \frac{dp}{dh}_{\text{GRAVITY}} + \frac{dp}{dh}_{\text{FRICTION}} + \frac{dp}{dh}_{\text{KINETIC}}$

1. Select $\Delta h$
2. Assume $\Delta p_i$
3. $p_{i+1} = p_i + \Delta p_i$
4. Calculate $H_L$, $\overline{\rho_m}$
5. Calculate $\Delta p$
6. $|\Delta p_i - \Delta p| \leq err$

- Yes
- No

Output $p_{i+1}$ at $h + \Delta h$
Finally, how to find $H_L$?

$$M = SG_o \cdot 350.52 \cdot \frac{1}{1 + WOR} + SG_w \cdot 350.52 \cdot \frac{WOR}{1 + WOR} + SG_g \cdot 0.0764 \cdot GLR$$

$$\rho_L = \frac{62.4 \cdot SG_o + \frac{R_s \cdot 0.0764 \cdot SG_g}{5.614} \cdot 1}{1 + WOR} + 62.4 \cdot SG_w \cdot \frac{WOR}{1 + WOR}$$

$$\rho_g = \frac{28.967 \cdot SG_g}{10.732 \cdot T_R}$$

$$\mu_L = \mu_o \cdot \frac{1}{1 + WOR} + \mu_w \cdot \frac{WOR}{1 + WOR}$$

$$\sigma_L = \sigma_o \cdot \frac{1}{1 + WOR} + \sigma_w \cdot \frac{WOR}{1 + WOR}$$

$$N_L = 0.15726 \cdot \mu_L \cdot \sqrt{\frac{1}{\rho_L g_L^3}}$$

$$N_{LV} = 1.938 \cdot v_{SL} \cdot \sqrt{\frac{\rho_L}{\sigma_L}}$$

$$N_{GV} = 1.938 \cdot v_{SG} \cdot \sqrt{\frac{\rho_L}{\sigma_L}}$$

$$N_D = 120.872 \cdot D \cdot \sqrt{\frac{\rho_L}{\sigma_L}}$$

$$H = \frac{N_{LV}}{N_{GV}^{0.875}} \cdot \left( \frac{p}{14.7} \right)^{0.1} \cdot \frac{CN_L}{N_D}$$

$$H_L = \sqrt{\frac{0.0047 + 1123.32H + 729489.64H^2}{1 + 1097.1566H + 722153.97H^2}}$$

$$B = \frac{N_{GV} N_{LV}^{0.38}}{N_D^{2.14}}$$

$$v_{SL} = \frac{5.615q_L}{86400 A_P} \left( B_o \frac{1}{1 + WOR} + B_w \frac{WOR}{1 + WOR} \right)$$

$$v_{SG} = \frac{q_L \left( GLR - R_s \left( \frac{1}{1 + WOR} \right) \right)}{86400 A_P} \cdot \frac{14.7 \cdot T_K}{p} \cdot \frac{z}{520}$$

Well, it’s a lot of equations :)

$$CN_L = 0.061 \cdot N_L^3 - 0.0929 \cdot N_L^2 + 0.0505 \cdot N_L + 0.0019$$

$$\psi = \begin{cases} 27170B^3 - 317.52B^2 + 0.5472B + 0.9999, & \text{if } B \leq 0.025 \\ -533.33B^2 + 58.524B + 0.1171, & \text{if } B > 0.025 \\ 2.5714B + 1.5962, & \text{if } B > 0.055 \end{cases}$$

$$H_L = \frac{H_L}{\psi} \times \psi$$
Summary

• Hagedorn and Brown correlation
• History and practical application
• Math & Physics
• Flow diagram to get the VLP curve
• Workflow to find $H_L$
What next?

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Thanks!